# Evolution of Perturbations in a Noncommutative Braneworld Inflation

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**Abstract**

Following our previous work in noncommutative braneworld inflation ( Ref. [18]), in this letter we use smeared, coherent state picture of noncommutativity to study evolution of perturbations in a noncommutative braneworld scenario. We show that in this setup, the early stage of the universe evolution has a phantom evolution with imaginary effective sound speed. We show also that the amplitude of perturbations in the commutative regime decays faster than the noncommutative regime with the same parameter values, and as a result we need smaller number of e-folds in the noncommutative regime to have a successful braneworld inflation.
**PACS**: 02.40.Gh, 11.10.Nx, 04.50.-h, 98.80.Cq
**Key Words**: Inflation, Perturbations, Noncommutativity

# 1 Introduction

Inspired by some aspects of string theory and loop quantum gravity, *fuzziness* of spacetime can be expressed using the following relation for non-commutativity of coordinate operators[1,2]

$$[\hat{x}^i, \hat{x}^j] = i\theta^{ij} \tag{1}$$

where $\theta^{ij}$ is a real, antisymmetric matrix, with the dimension of length squared which determines the fundamental cell discretization of spacetime manifold. As a consequence of above relation, the notion of point in the spacetime manifold becomes obscure as there is a fundamental uncertainty in measuring the coordinates

$$\Delta x^i \Delta x^j \geq \frac{1}{2}|\theta^{ij}|. \tag{2}$$

This finite resolution of the spacetime points especially affects the cosmological dynamics in early stages of the universe evolution. On the other hand, inflation has been identified as a great opportunity to test theories of Planck scale physics including noncommutative geometry. Essentially, effects of trans-Planckian physics should be observable in the cosmic microwave background radiation [3-9]. For this reason, various attempts to construct noncommutative inflationary models have been done by adopting various different approaches. These approaches include using relation (1) for space-space [10] and space-time [11] coordinates and constructing a noncommutative field theory on the spacetime manifold by replacing ordinary product of fields by Weyl-Wigner-Moyal $*-$*product*. Another way to incorporate effects of high energy physics in inflationary models is using the generalized uncertainty principle (GUP) which is a manifestation of the existence of a fundamental length scale in the system [12].

Recently a new approach to noncommutative inflation has been proposed by Rinaldi [13] using the coherent state picture of noncommutativity introduced in [14]. This model is free from some of the problems that plagued models based on $*-$*product* such as unexpected divergencies and UV/IR mixing (see [15] for a full review). The key idea in this model is that noncommutativity *smears* the initial singularity and as a result there will be an smooth transition between pre and post big bang eras via an accelerated expansion. It has been shown that noncommutativity eliminates point-like structures in the favor of smeared objects in flat spacetime. As Nicolini *et al.* have shown [16] ( see also [17] for some other extensions ), the effect of smearing is mathematically implemented as a substitution rule: position Dirac-delta function is replaced everywhere with a Gaussian distribution of minimal width $\sqrt{\theta}$. In this framework, they have chosen the mass density of a static, spherically symmetric, smeared, particle-like gravitational source as follows

$$\rho_\theta(r) = \frac{M}{(2\pi\theta)^{\frac{3}{2}}} \exp(-\frac{r^2}{4\theta}). \tag{3}$$

As they have indicated, the particle mass $M$, instead of being perfectly localized at a point, is diffused throughout a region of linear size $\sqrt{\theta}$. This is due to the intrinsic uncertainty as has been shown in the coordinate commutators (1).

Recently we have constructed a noncommutative braneworld inflation scenario [18] based on the idea that initial singularity is smeared in a noncommutative background. Within the same streamline, the purpose of this letter is to study time evolution of cosmological perturbations in a braneworld inflation scenario in the context of spacetime noncommutativity.

## 2 Cosmological dynamics in the noncommutative RS II model

The 5D field equations in the Randall-Sundrum (RS) II [19] setup are

$$ {}^{(5)}G_{AB} = -\Lambda_5\, {}^{(5)}g_{AB} + \delta(y)\,\frac{8\pi}{M_5^3}\left[-\lambda g_{AB} + T_{AB}\right], \tag{4} $$

where $y$ is a Gaussian normal coordinate orthogonal to the brane ( the brane is localized at $y = 0$), $\lambda$ is the brane tension, and $T_{AB}$ is the energy-momentum tensor of particles and fields confined to the brane . The effective field equations on the brane are derived from the Gauss-Codazzi equations and junction conditions (using $Z_2$-symmetry)[20,21]

$$ G_{ab} = -\Lambda g_{ab} + \kappa^2 T_{ab} + 6\frac{\kappa^2}{\lambda}\mathcal{S}_{ab} - \mathcal{E}_{ab}\ , \tag{5} $$

where $\mathcal{S}_{ab} \sim (T_{ab})^2$ is the high-energy correction term, which is negligible for $\rho \ll \lambda$, while $\mathcal{E}_{ab}$ is the projection of the bulk Weyl tensor on the brane. The general form of the brane energy-momentum tensor for any matter fields (scalar fields, perfect fluids, kinetic gases, dissipative fluids, etc.), including a combination of different fields, can be covariantly given in terms of a chosen 4-velocity $u^\mu$ as

$$ T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} + \pi_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu\ . \tag{6} $$

Here $\rho$ and $p$ are the energy density and isotropic pressure and $q$ and $\pi$ are the momentum density and anisotropic stress respectively. $h_{\mu\nu}$ defined as

$$ h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu = {}^{(5)}g_{\mu\nu} - n_\mu n_\nu + u_\mu u_\nu \tag{7} $$

projects into the comoving rest space at each event where $n_\nu$ is the spacelike unit normal to the brane. The modified Friedmann and Raychaudhuri equations in the background are [20]

$$ H^2 = \frac{\kappa^2}{3}\rho\left(1+\frac{\rho}{2\lambda}\right) + \frac{C}{a^4} + \frac{1}{3}\Lambda - \frac{K}{a^2}\,, \tag{8} $$

and

$$ \dot{H} = -\frac{\kappa^2}{2}(\rho+p)\left(1+\frac{\rho}{\lambda}\right) - 2\frac{C}{a^4} + \frac{K}{a^2}\,, \tag{9} $$

respectively. By definition, $C = \frac{\kappa^2}{3}\rho_{\varepsilon 0}a_0^4$ where $\rho_{\varepsilon 0}$ is the dark radiation energy density. For a matter content consisted of a perfect fluid or a minimally coupled scalar field the total effective energy density, pressure, momentum density and anisotropic stress can be written as [21]

$$ \begin{aligned} \rho^{\text{eff}} &= \rho\left(1+\frac{\rho}{2\lambda}+\frac{\rho^\varepsilon}{\rho}\right)\ , && (10)\\ p^{\text{eff}} &= p + \frac{\rho}{2\lambda}(2p+\rho) + \frac{\rho^\varepsilon}{3}\ , && (11)\\ q_a^{\text{eff}} &= q_a^\varepsilon\ , && (12)\\ \pi_{ab}^{\text{eff}} &= \pi_{ab}^\varepsilon\,, && (13) \end{aligned} $$

where superscript $\varepsilon$ denotes the contribution of the bulk Weyl tensor which enters the modified friedmann equation as a non-local dark radiation term. Using these definitions, the modified Friedmann and Raychaudhuri equations can be rewritten as

$$\begin{aligned} H^2 &= \frac{\kappa^2}{3}\rho^{\text{eff}} + \frac{1}{3}\Lambda + \frac{K}{a^2}\,, \qquad (14)\\ \dot{H} &= -\frac{\kappa^2}{2}(\rho^{\text{eff}} + p^{\text{eff}}) + \frac{K}{a^2}\,. \qquad (15)\end{aligned}$$

The tracefree property of $\mathcal{E}^{\mu}_{\nu}$ in equation (5) implies that the pressure obeys $P^{\varepsilon} = \frac{1}{3}\rho^{\varepsilon}$. The local conservation equations on the brane are [21]

$$\dot{\rho} + \Theta(\rho + p) = 0\,, \qquad (16)$$
$$D_a p + (\rho + p)A_a = 0 \qquad (17)$$

where $\Theta$ is the volume expansion rate, which reduces to $3H$ in the FRW background ($H$ is the background Hubble rate), $A_a$ is the 4-acceleration, and $D_a$ is the covariant derivative in the rest space. The non-local conservation equations for the dark radiation matter can be expressed as [21]

$$\dot{\rho}^{\varepsilon} + \frac{4}{3}\Theta\rho^{\varepsilon} + D^a q^{\varepsilon} = 0 \qquad (18)$$
$$\dot{q}^{\varepsilon}_a + 4Hq^{\varepsilon}_a + \frac{1}{3}D_a\rho^{\varepsilon} + \frac{4}{3}\rho^{\varepsilon}A_a + D^b\pi^{\varepsilon}_{ab} = -\frac{(\rho + p)}{\lambda}D_a\rho\,. \qquad (19)$$

We now suppose that the initial singularity that leads to RS II geometry afterwards, is smeared due to spacetime noncommutativity. A newly proposed model for the similar scenario in the usual 4D universe suggests that one could write the energy density as [13,18]

$$\rho(t) = \frac{1}{32\pi^2\theta^2}e^{-t^2/4\theta}\,. \qquad (20)$$

Note that we suppose that the universe enters the RS II geometry immediately after the initial smeared singularity which is a reasonable assumption (for instance, from a M-theory perspective of the cyclic universe this assumption seems to be reliable, see Ref. [22]). Using equation (20), and setting $\Lambda = 0 = K$, the Friedmann equation (14) in noncommutative space could be rewritten as follows

$$H^2 = \frac{\kappa^2}{3}\rho^{\text{eff}}(t) \qquad (21)$$

where $\rho^{\text{eff}}$ is given by equation (10). From equation (16) one can find the effective noncommutative pressure using equation (20) as

$$p = -\rho + \frac{t}{6\theta}e^{-t^2/8\theta}\,. \qquad (22)$$

So, the equation of state parameter will be

$$\omega = -1 + \frac{16}{3}\pi^2\theta\, te^{-t^2/8\theta} \qquad (23)$$

and the speed of sound is

$$c_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{-3t - 64\theta^2\pi^2 e^{-t^2/8\theta} + 32\theta\pi^2 t^2 e^{-t^2/8\theta}}{3t}\,. \tag{24}$$

Using equations (10) and (11) we can find the *effective* equation of state and speed of sound. To do this end, we note that there are constraints from nucleosynthesis on the value of $\rho^{\varepsilon}$ so that $\frac{\rho^{\varepsilon}}{\rho} \leq 0.03$ at the time of nucleosynthesis [23,24]. In this respect, we can neglect this contribution to find

$$\begin{aligned}\omega^{\rm eff} = \frac{1}{192}\,{\rm e}^{-\frac{1}{8}\frac{t^2}{\theta}}\Big[-192\,\pi^2\theta^2\lambda + 1024\,t{\rm e}^{\frac{-t^2}{8\theta}}\pi^4\theta^3\lambda - 3\,{\rm e}^{\frac{-t^2}{8\theta}} + 32\,t{\rm e}^{\frac{-t^2}{4\theta}}\pi^2\theta\Big]\times \\ \left[\theta\left(\frac{1}{64}\left(64\,\pi^2\theta^2\lambda + {\rm e}^{\frac{-t^2}{8\theta}}\right)\pi^{-2}\theta^{-2}\lambda^{-1}\right)\right]^2\times \\ \pi^{-2}\theta^{-4}\lambda^{-1}\left[{\rm e}^{\frac{-t^2}{8\theta}}\left(\frac{1}{64}\left(64\,\pi^2\theta^2\lambda + {\rm e}^{\frac{-t^2}{8\theta}}\right)\pi^{-2}\theta^{-2}\lambda^{-1}\right)\right]^{-1}\end{aligned} \tag{25}$$

which simplifies to the following equation for the high energy regime ($\rho \gg \lambda$)

$$\omega^{\rm eff} \approx -1 + \frac{32}{3}\pi^2\theta\, te^{-t^2/8\theta}\,. \tag{26}$$

Similarly, the effective speed of sound in the high energy regime will be

$$(c_s^2)^{\rm eff} \approx \frac{16}{3}\pi^2\theta\, te^{-t^2/8\theta} + \frac{-3t - 64\theta^2\pi^2 e^{-t^2/8\theta} + 32\theta\pi^2 t^2 e^{-t^2/8\theta}}{3t}\,. \tag{27}$$

Figure 1$a$ shows the evolution of the equation of state parameter and the effective equation of state parameter as given by equations (23) and (25) respectively. As one can see from this figure, there is a small variation in $\omega$ and $\omega^{eff}$ around the smeared singularity. Figure 1$b$ shows the evolution of the effective speed of sound. It is obvious from this figure that in $t > 0$ and in the high energy noncommutative regime, $c_s$ is imaginary. In this respect, the evolution of the universe in the early, inflationary stage is a phantom evolution.

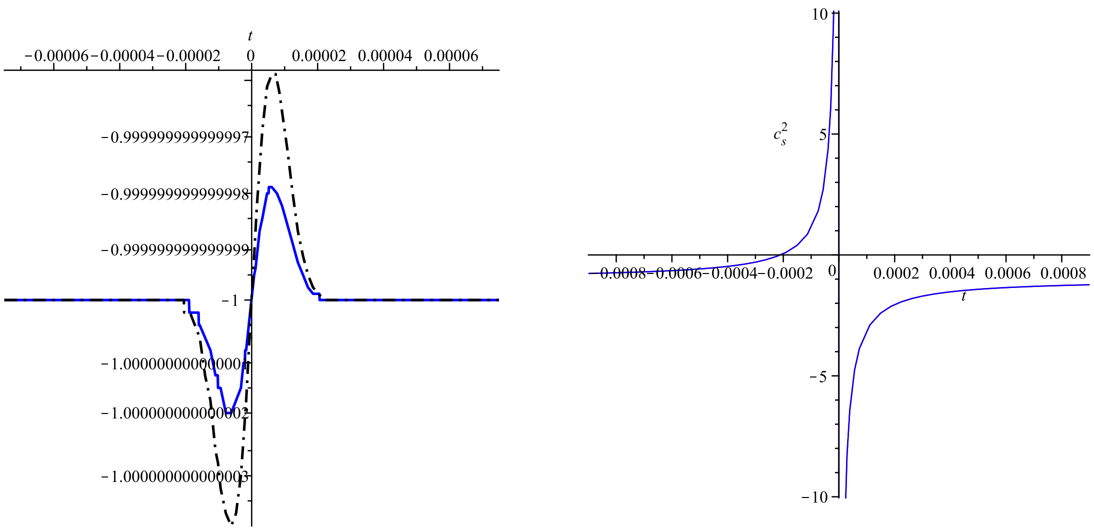


Figure 1: a) Evolution of the noncommutative equation of state parameter (solid line) and the noncommutative effective equation of state parameter ( dashed line) versus the cosmic time. b) Evolution of the noncommutative effective speed of sound versus the cosmic time. For $t > 0$ and in the high energy noncommutative regime, $c_s$ is imaginary ( a phantom evolution).

We use these results in the next section to determine time evolution of the cosmological perturbations.

# 3 Evolution of large scale scalar perturbations

The evolution of cosmological perturbations in the Randall-Sundrum braneworld scenario has been studied extensively ( see for instance [25] and references therein). To analyze the scalar perturbations in our noncommutative setup, we define energy density and expansion perturbations as [26] using the covariant 3+1 analysis developed in [27]

$$\Delta = \frac{a^2}{\rho} D^2 \rho \,, \qquad Z = a^2 D^2 \Theta \,. \tag{28}$$

Similarly, the perturbations in nonlocal quantities associated with the dark radiation matter are defined as

$$U = \frac{a^2}{\rho} D^2 \rho^{\varepsilon} \,, \qquad Q = \frac{a}{\rho} D^2 q^{\varepsilon} \,, \qquad \Pi = \frac{1}{\rho} D^2 \pi^{\varepsilon} \,. \tag{29}$$

With these definitions, the equations governing on the evolution of perturbations (equations (16)-(19)) will take the following forms

$$\dot{\Delta} = 3wH\Delta - (1+w)Z \,, \tag{30}$$

$$\dot{Z} = -2HZ - \left(\frac{c_{\rm s}^2}{1+w}\right) D^2 \Delta - \kappa^2 \rho U - \frac{1}{2}\kappa^2 \rho \left[1 + (4+3w)\frac{\rho}{\lambda} - \left(\frac{4c_{\rm s}^2}{1+w}\right)\frac{\rho^{\varepsilon}}{\rho}\right] \Delta \,, \tag{31}$$

$$
\begin{aligned}
\dot{U} &= (3w-1)HU + \left(\frac{4c_{\mathrm{s}}^2}{1+w}\right)\left(\frac{\rho^\varepsilon}{\rho}\right)H\Delta - \left(\frac{4\rho^\varepsilon}{3\rho}\right)Z - aD^2Q\,, &(32)\\
\dot{Q} &= (3w-1)HQ - \frac{1}{3a}U - \frac{2}{3}aD^2\Pi + \frac{1}{3a}\left[\left(\frac{4c_{\mathrm{s}}^2}{1+w}\right)\frac{\rho^\varepsilon}{\rho} - 3(1+w)\frac{\rho}{\lambda}\right]\Delta\,. &(33)
\end{aligned}
$$

where $\rho$ , $\omega$ and $c_s$ are given by equations (20) , (23) and (24) respectively.
In general, scalar perturbations on the brane cannot be predicted by brane observers without additional information from the bulk because there is no equation for $\dot{\Pi}$ in the above set of equations. Nevertheless, it has been shown that on large scales one can neglect the $D^2\Pi$ term in equation (33). So, on large scales, the system of equations closes on the brane, and brane observers can predict scalar perturbations from initial conditions intrinsic to the brane without the need to solve the bulk perturbation equations [26,27].
To solve the above system of equations using the simplification mentioned, we introduce two new variables; the first is a scalar covariant curvature perturbation variable

$$
C \equiv a^4D^2R = -4a^2HZ + 2\kappa^2a^2\rho\left(1+\frac{\rho}{2\lambda}\right)\Delta + 2\kappa^2a^2\rho U\,, \tag{34}
$$

where $R$ is the Ricci curvature of the surfaces orthogonal to $u^\mu$. The second variable is a covariant analog of the Bardeen metric potential $\Phi_H$,

$$
\Phi = \kappa^2a^2\rho\Delta\,. \tag{35}
$$

Along each fundamental world-line, covariant curvature perturbation, $C$, is locally conserved

$$
C = C_0\,, \quad \dot{C}_0 = 0\,. \tag{36}
$$

With these new variables, the system of equations reduces to

$$
\begin{aligned}
\dot{\Phi} &= -H\left[1+(1+w)\frac{\kappa^2\rho}{2H^2}\left(1+\frac{\rho}{\lambda}\right)\right]\Phi - \left[(1+w)\frac{a^2\kappa^4\rho^2}{2H}\right]U + \left[(1+w)\frac{\kappa^2\rho}{4H}\right]C_0 &(37)\\
\dot{U} &= -H\left[1-3w+\frac{2\kappa^2\rho^\varepsilon}{3H^2}\right]U - \frac{2\rho^\varepsilon}{3a^2H\rho}\left[1+\frac{\rho}{\lambda} - \frac{6c_{\mathrm{s}}^2H^2}{(1+w)\kappa^2\rho}\right]\Phi + \left[\frac{\rho^\varepsilon}{3a^2H\rho}\right]C_0\,. &(38)
\end{aligned}
$$

If there is no dark radiation in the background, $\rho^\varepsilon = 0$, then

$$
U = U_0\exp\left\{\int(3w-1)dN\right\}, \tag{39}
$$

where $N$ is the number of e-folds. In this case, the above system reduces to a single equation for $\Phi$ which is

$$
\frac{d\Phi}{dN} + \left[1+\frac{(1+w)\kappa^2\rho}{2H^2}\left(1+\frac{\rho}{\lambda}\right)\right]\Phi = \left[\frac{(1+w)\kappa^2\rho}{4H^2}\right]C_o - \left[\frac{3(1+w)a_o^2\rho^2}{\lambda H^2}\right]e^{2N}U \tag{40}
$$

where $U$ is given by (39). We use these results in the next section to study noncommutative modifications of the scalar perturbations dynamics.

# 4 Noncommutative modifications

Now we want to solve equation (40) using explicit noncommutative forms of $\rho$, $H$, $\omega$ and $U$ given by (20), (21) , (23) and (39) respectively. To do this end, we need to specify the noncommutative form of $N$ which has been appeared in equation (39). As we have shown in Ref. [18], the noncommutative number of e-folds is given by

$$N=\int_{t_i}^{t_f} H dt \simeq \frac{8}{3}\pi\kappa^2\,\rho_0\left[\sqrt{\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\frac{t_f}{\sqrt{\theta}}\Big)+\frac{1}{2}\sqrt{2\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\frac{\sqrt{2}t_f}{\sqrt{\theta}}\Big)\lambda^{-1}\right]$$
$$-\frac{8}{3}\pi\kappa^2\,\rho_0\left[\sqrt{\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\frac{t_i}{\sqrt{\theta}}\Big)+\frac{1}{2}\sqrt{2\pi\theta}\;\operatorname{erf}\Big(\frac{1}{2}\frac{\sqrt{2}t_i}{\sqrt{\theta}}\Big)\lambda^{-1}\right] \tag{41}$$

where erf(x) denotes the error function. By expanding the error functions in equation (41) in series, the number of e-folds (supposing that the universe enters the inflationary phase immediately after the big bang, that is, $t_i=0$ and $t_f=t$) will be given by

$$N\simeq\frac{8}{3}\pi\kappa^2\,\rho_0\left[t-\frac{1}{12}\frac{t^3}{\sqrt{\pi}\theta^{\frac{3}{2}}}+\frac{1}{160}\frac{t^5}{\sqrt{\pi}\theta^{\frac{5}{2}}}+\frac{1}{2}\Big(2\,t-\frac{1}{6}\frac{\sqrt{2}t^3}{\sqrt{\pi}\theta^{\frac{3}{2}}}+\frac{1}{40}\frac{\sqrt{2}t^5}{\sqrt{\pi}\theta^{\frac{5}{2}}}\Big)\lambda^{-1}\right]. \tag{42}$$

Now we can integrate equation (40) to find

$$\Phi=\frac{1}{2}(1+\omega)\frac{\rho\,\lambda\,\kappa^2C_0}{2H^2\lambda+(1+\omega)(\kappa^2\rho\lambda+\kappa^2\rho^2)}$$
$$-6(1+\omega)\frac{H^2\rho\,\lambda\,a_0{}^2U\exp\left(3\,\omega\,a-3\,\omega\,a_0-a+a_0\right)}{6H^2\lambda+(1+\omega)(\kappa^2\rho\lambda+\kappa^2\rho^2)}\exp\Big(\frac{-t^2}{4\theta}\Big)$$
$$+\exp\left[-\frac{1}{2}\frac{2\,H^2\lambda+(1+\omega)(\kappa^2\rho\,\lambda+\kappa^2\rho^2)}{H^2\lambda}\frac{t^2}{8\theta}\right]. \tag{43}$$

Figure (2) shows the evolution of $\Phi$ for both usual braneworld scenario and our noncommutative setup in the high energy inflation regime ($\rho\gg\lambda$). One should note that the subsequent evolution of the universe after times greater than a few $\sqrt{\theta}$ should be governed by a matter content[1] different than the one used in equation (21) (*i.e.* energy density of the initial singularity smeared by noncommutativity). So, the evolution of $\omega$, $c_s^2$ and $\Phi$ in the low energy regime essentially should be different. The solution (43) is valid when there is no dark radiation in the background. If $\rho^\varepsilon\neq 0$, then one should solve the system of equations (37) and (38) using the explicit form of $\rho^\varepsilon$. Generally the time dependence of $\rho^\varepsilon$ for brane observer is not determined. Here we introduce a possible candidate for this quantity: as we have mentioned previously, the constraint from nucleosynthesis on the value of $\rho^\varepsilon$ is so that $\frac{\rho^\varepsilon}{\rho}\leq 0.03$ at the time of nucleosynthesis. Based on this constraint, we can assume for instance that $\rho^\varepsilon$ is a small fraction of $\rho$ at a given time. Since the time evolution of $\rho$ is determined by (20), the time evolution of $\rho^\varepsilon$ can be supposed to be

$$\rho^\varepsilon(t)=\frac{\delta}{32\pi^2\theta^2}e^{-t^2/4\theta}$$

[1] See for instance [28] for particle creation in an expanding universe.

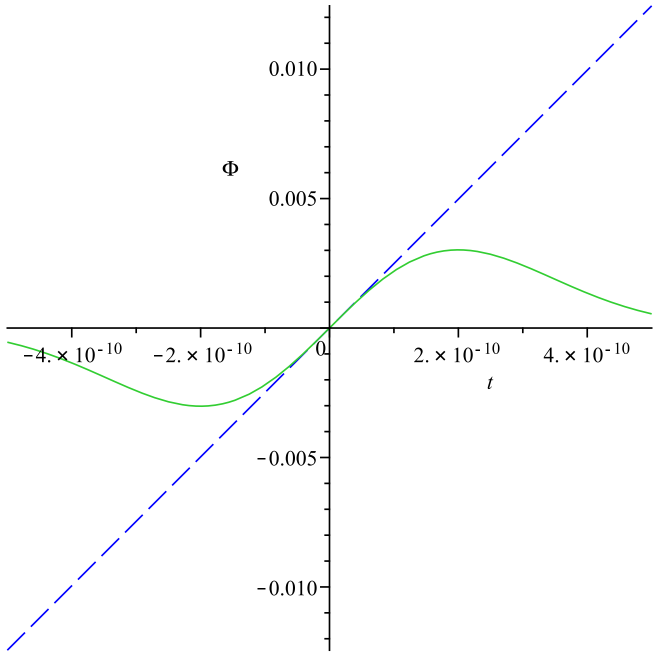


Figure 2: Evolution of the parameter $\Phi$ which is an analog of the Bardeen metric potential as defined in (35) for both usual braneworld scenario (dashed line) and our noncommutative setup (solid line) when $\frac{\rho_0}{\lambda} = 10^{10}$. We assumed that no dark radiation is present in the background geometry.

where $\delta$ is a small constant and less than 0.03. This form of $\rho^{\varepsilon}(t)$ can be used to solve the system of equations (37) and (38) explicitly. Nevertheless, this procedure needs a lot of calculations with very lengthy solutions that we ignore their presentation here.
The curvature perturbation defined in metric-based perturbation theory is

$$\xi = \mathcal{R} + \frac{\delta\rho}{3(\rho+p)}\,, \tag{44}$$

which reduces to $\mathcal{R}$ on uniform density ( $\delta\rho = 0$ ) hypersurfaces. If there is no dark radiation in the background ($\rho^{\varepsilon} = 0$), the total curvature perturbation on large scale is given by the following differential equation [24]

$$\dot{\xi}^{\,\mathrm{eff}} = \dot{\xi}^{\,\mathrm{m}} + H\left[c_{\mathrm{s}}^2 - \frac{1}{3} + \left(\frac{\rho+p}{\rho+\lambda}\right)\right]\frac{\delta\rho^{\varepsilon}}{(\rho+p)(1+\rho/\lambda)}\,. \tag{45}$$

where $\xi^m$ is matter perturbation which is zero for adiabattic perturbations. Since the time variation of $\rho,\ H,\ p,\ c_s$ and $\delta\rho^{\varepsilon}$ are given by equations (20), (21), (22), (24) and (39) respectively, we can obtain the time evolution of the curvature perturbation explicitly as follows

$$\xi^{\,\mathrm{eff}} = \frac{1}{96}\frac{\mathrm{Ei}\left(1, \frac{3t^2}{8\theta}\right)}{\pi^2\theta^2\lambda} - \frac{1}{48}\frac{\mathrm{e}^{-\frac{3t^2}{8\theta}}}{\pi^2\theta^2\lambda} + \frac{1}{3}\,\mathrm{Ei}\left(1, \frac{t^2}{4\theta}\right) - \frac{2}{3}\,\mathrm{e}^{-\frac{t^2}{4\theta}}$$
$$-\frac{1}{768}\,\mathrm{erf}\left(\frac{t}{2\sqrt{\theta}}\right)\lambda^{-1}\pi^{-3}\theta^{-3}\frac{1}{\sqrt{\theta\,\pi}} - \frac{1}{24}\,\mathrm{erf}\left(\frac{1}{4}\,\frac{\sqrt{2}t}{\sqrt{\theta}}\right)\sqrt{2}\theta^{-1}\pi^{-1}\frac{1}{\sqrt{\theta\,\pi}} \tag{46}$$

Where $\mathrm{Ei}(a,z)$ is the exponential integral defined as $\mathrm{Ei}(a,z) = z^{a-1}\Gamma(1-a,z)$. Figure 3 shows the evolution of $\xi^{\,\mathrm{eff}}$ versus the cosmic time for both commutative and noncommutative brane inflation. For the commutative braneworld inflation, we have considered the chaotic inflation with $V(\phi) = \frac{1}{2}m^2\phi^2$ and we obtained the form of $\phi$ using its relation with $\rho$. The amplitude of perturbations in the commutative regime decays faster than the noncommutative regime with the same parameter values. As a result we need smaller number of e-folds in the noncommutative regime to have a successful braneworld inflation.

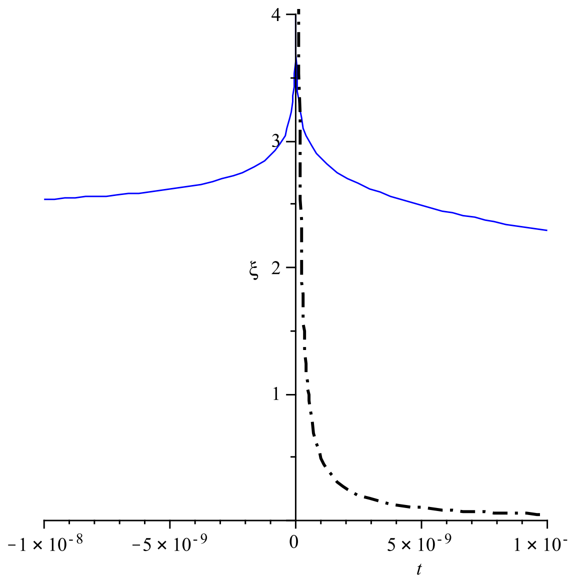


Figure 3: a) Evolution of the parameter $\xi$ in commutative brane inflation ( dashed line). b) Evolution of the parameter $\xi$ defined in (44)( with the same parameters values as in figure (2)) when $\frac{\rho_0}{\lambda} = 10^{10}$ and no dark radiation is present in the background geometry (solid lines).

# 5 Conclusion

Spacetime noncommutativity as a trans-Planckian effect, essentially could have some observable effects on the cosmic microwave background radiation. In this respect, it is desirable to study an inflation scenario within a noncommutative background. Recently we have shown the possibility of realization of a non-singular, bouncing, early time cosmology in a noncommutative braneworld scenario [18]. In that work, using the smeared, coherent state picture of the spacetime noncommutativity, we have constructed a braneworld inflation that has the potential to support the scale invariant scalar perturbations. Here, following our previous work, we have studied the time evolution of the perturbations in this noncommutative braneworld setup. We have neglected the contribution of the dark radiation term ( originating in the bulk Weyl tensor) in the background geometry to have a closed set of equations on the brane. However, the contribution of this term in the evolution of perturbations on the brane are taken into account. In this way, by studying the effective quantities ( such as the effective equation of state and speed of sound ) we have realized the possibility of a

phantom evolution in the early, inflationary stage of the universe history. Our analysis of the perturbations on the brane shows that the amplitude of perturbations in the commutative regime decays faster than the noncommutative regime with the same parameter values. As a result we need smaller number of e-folds in the noncommutative regime to have a successful braneworld inflation.

**Acknowledgment**
This work has been supported partially by Research Institute for Astronomy and Astrophysics of Maragha, IRAN.

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